Typical correction for Physics 2 remedial exam (2024)

Course questions:

1. C **2. B** **3. A** **4. B** **5. C**

1 1 1 1 1

Exercise 1:.

1. The expressions for the electric field and potential:
   a. In the region between the two sphere and the shell ($R_A < r < R_i$);

Using the Gauss theorem:

$$\oiint \vec{E} \cdot \overrightarrow{ds} = \frac{Q_A}{\varepsilon_0} \Rightarrow E 4\pi r^2 = \frac{Q_A}{\varepsilon_0} \Rightarrow E = \frac{Q_A}{4\pi\varepsilon_0 r^2} = \frac{kQ_A}{r^2} \quad 1 \quad for \quad R_A < r < R_i$$

Using the superposition principal for the sphere and the shell: (see course chapter 5)

$$V = \frac{kQ_A}{r} + \frac{kQ_i}{R_i} + \frac{kQ_e}{R_e} \quad 0,5$$

We have: $V(R_A) = 0$ (sphere A is linked to the earth)

$$\frac{kQ_A}{R_A} + \frac{kQ_i}{R_i} + \frac{kQ_e}{R_e} = 0 \Rightarrow \frac{kQ_i}{R_i} + \frac{kQ_e}{R_e} = -\frac{kQ_A}{R_A} \quad 0,8$$

Then

$$V = \frac{kQ_A}{r} - \frac{kQ_A}{R_A} = kQ_A\left(\frac{1}{r} - \frac{1}{R_A}\right) \quad for \quad R_A < r < R_i$$

1

   b. Outside B ($r > R_e$).

Using the Gauss theorem:

$$\oiint \vec{E} \cdot \overrightarrow{ds} = \frac{Q_A + Q_i + Q_e}{\varepsilon_0} \Rightarrow E 4\pi r^2 = \frac{Q_A + Q_i + Q_e}{\varepsilon_0} \Rightarrow E = \frac{Q_A + Q_i + Q_e}{4\pi\varepsilon_0 r^2} = \frac{k(Q_A + Q_i + Q_e)}{r^2} \quad 0,5$$

We have $Q_A = -Q_i$ (because of the total influence), then

$$E = \frac{kQ_e}{r^2} \quad for \quad r > R_e \quad 1$$

Using the superposition principal for the sphere and the shell: (see course chapter 5)

$$V = \frac{kQ_e}{r} + \frac{kQ_i}{r} + \frac{kQ_e}{r} = \frac{k(Q_A + Q_i + Q_e)}{r} = \frac{kQ_e}{r} \quad for\ r > R_e \quad 0,8$$

2. Calculate Q_A, Q_i and Q_e

$$\frac{kQ_e}{R_e} = V_B \Rightarrow Q_e = \frac{R_e V_B}{k} = \frac{0.9 \times 1000}{9 \times 10^9} = 10^{-7} C \quad 1$$

The shell carries no net charge:

$$Q_i + Q_e = 0 \Rightarrow Q_i = -Q_e = -10^{-7} C$$

$$Q_A = -Q_i = 10^{-7} C \quad 1$$

Exercise 2:.

The electric potential at P due to the charge element dq of the ring is given by:

$$dV = \frac{kdq}{r} = \frac{k\lambda a d\theta}{\sqrt{a^2+x^2}}$$ 1

Hence, the electric potential at P due to the uniformly charged ring is given by:

$$V = \int_0^{2\pi} \frac{k\lambda a d\theta}{\sqrt{a^2+x^2}} = \frac{k\lambda a}{\sqrt{a^2+x^2}}\int_0^{2\pi} d\theta = \frac{k\lambda a 2\pi}{\sqrt{a^2+x^2}}$$ 1

$$E = -\vec{\nabla}V = -\frac{\partial V}{\partial x}\vec{e}_x = \frac{k\lambda a 2\pi x}{(a^2+x^2)^{3/2}}\vec{e}_x$$ 1

Exercise 3:.

- The matrix equations:

By Applying the mesh current method to the network of Fig. 3, we can write the system of equations:

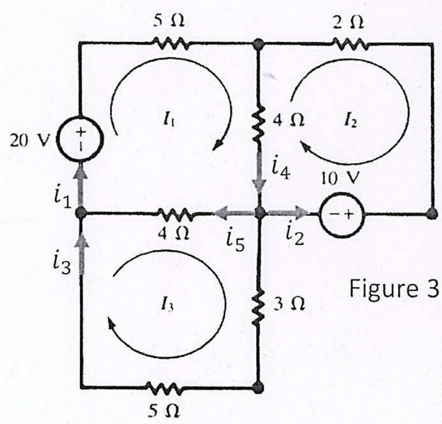
$$\begin{cases} 20 - 5I_1 - 4(I_1 - I_2) - 4(I_1 - I_3) = 0 \\ -10 - 4(I_2 - I_1) - 2I_2 = 0 \\ -4(I_3 - I_1) - 8I_3 = 0 \end{cases} \begin{cases} 13I_1 - 4I_2 - 4I_3 = 20 \\ 4I_1 - 6I_2 - 0I_3 = 10 \\ 4I_1 + 0I_2 - 12I_3 = 0 \end{cases}$$ 1 1 1

$$\Delta_I = \begin{vmatrix} 13 & -4 & -4 \\ 4 & -6 & 0 \\ 4 & 0 & -12 \end{vmatrix} = 13\big((-6)\times(-12)\big) + 4\big(4\times(-12)\big) + 4\big((-6)\times 4\big) = 648$$ 0,25

$$I_1 = \frac{\begin{vmatrix} 20 & -4 & -4 \\ 10 & -6 & 0 \\ 0 & 0 & -12 \end{vmatrix}}{\Delta_I} = \frac{960}{648} = 1.48A$$ 0,25

$$I_2 = \frac{\begin{vmatrix} 13 & 20 & -4 \\ 4 & 10 & 0 \\ 4 & 0 & -12 \end{vmatrix}}{\Delta_I} = \frac{-440}{648} = -0.68A$$ 0,25

$$I_3 = \frac{\begin{vmatrix} 13 & -4 & 20 \\ 4 & -6 & 10 \\ 4 & 0 & 0 \end{vmatrix}}{\Delta_I} = \frac{320}{648} = 0.49A$$ 0,25



0,5

Figure 3

- The courant of each branch: (see figure 3)

$i_1 = I_1 = 1.48A,$ 0,25 $\quad i_2 = -I_2 = 0.68A,$ 0,25 $\quad i_3 = I_3 = 0.49A,$ 0,25 $\quad i_4 = I_1 - I_2 = 2.17A,$ 0,25 $\quad i_5 = I_1 - I_3 = 0.99A$ 0,25